UNIVERSITY OF TWENTE.

Importance sampling for dynamic fault trees

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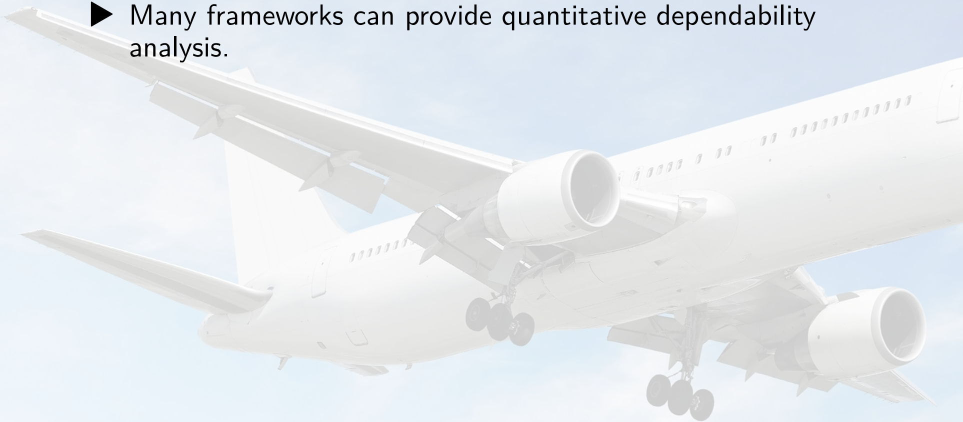
Stoelinga

University of Twente

18 October 2018

Our contribution in a nutshell

- Many frameworks can provide quantitative dependability analysis.



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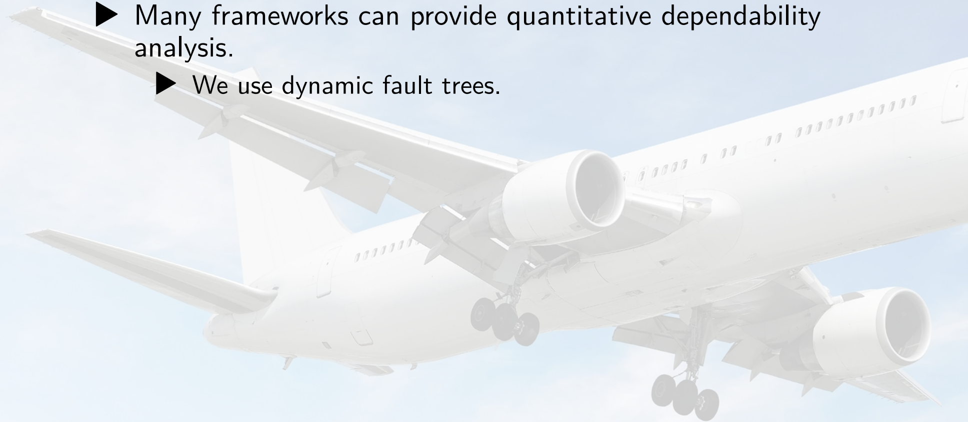

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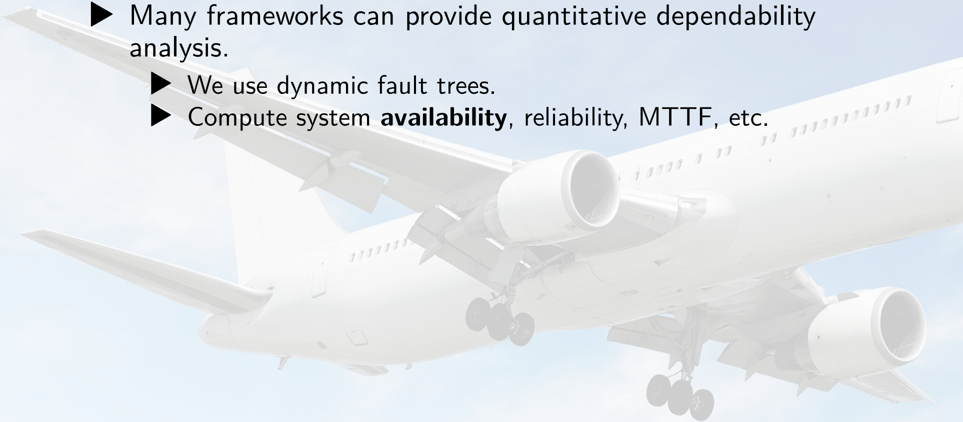

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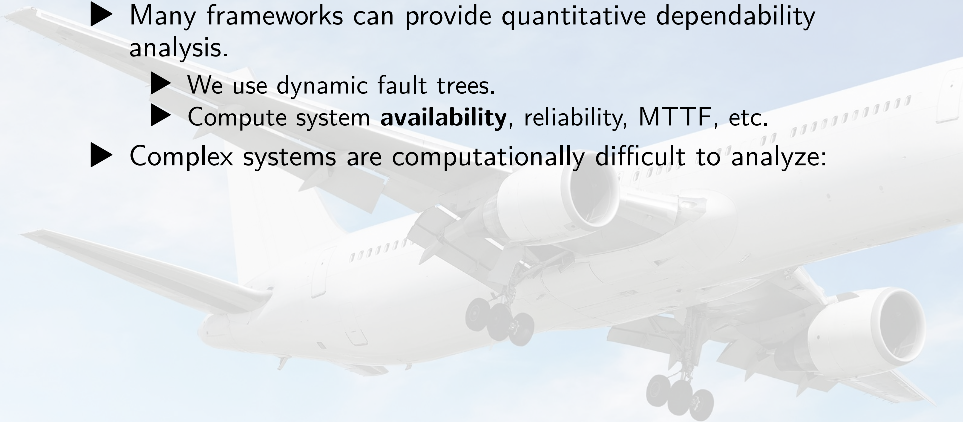

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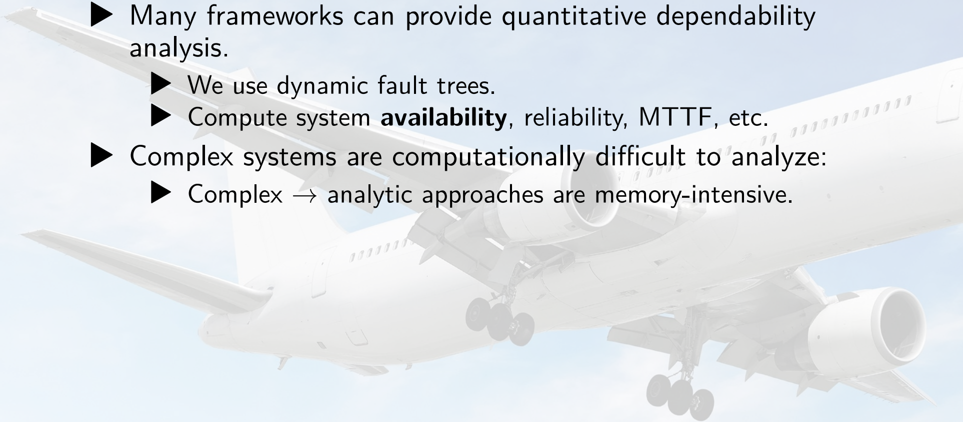

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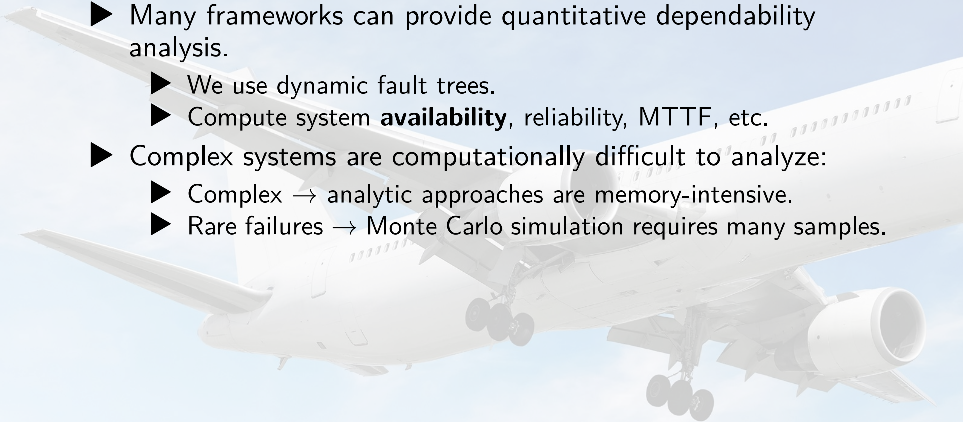

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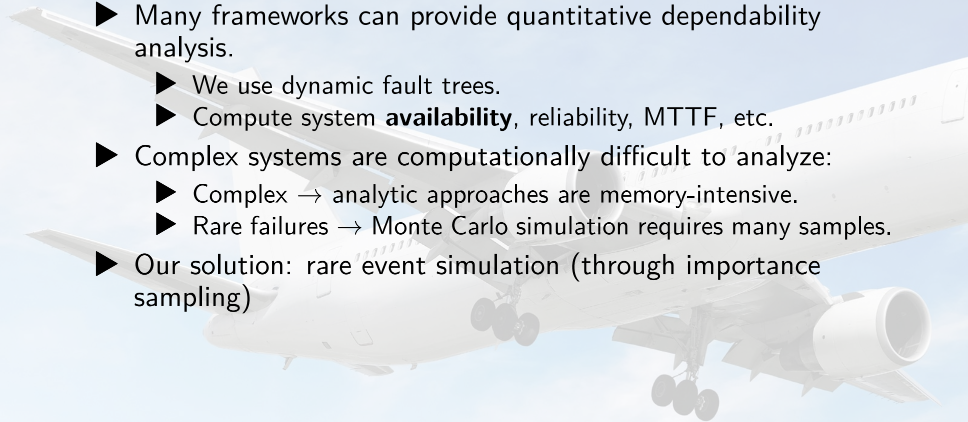

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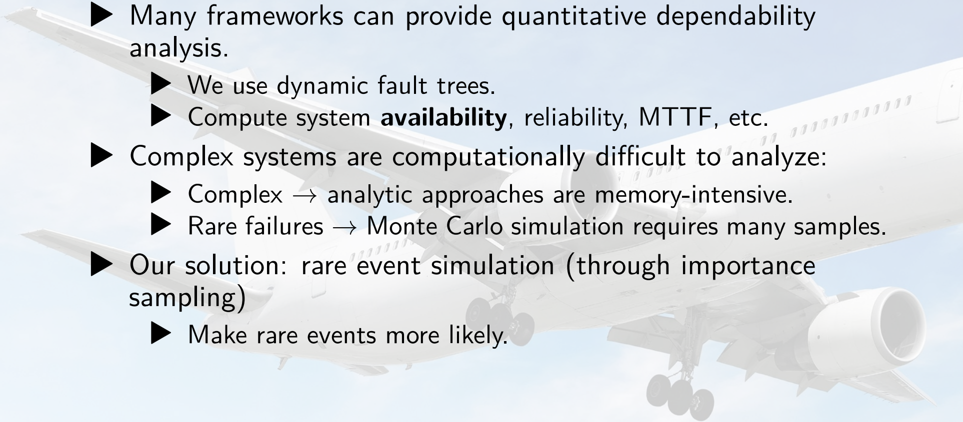

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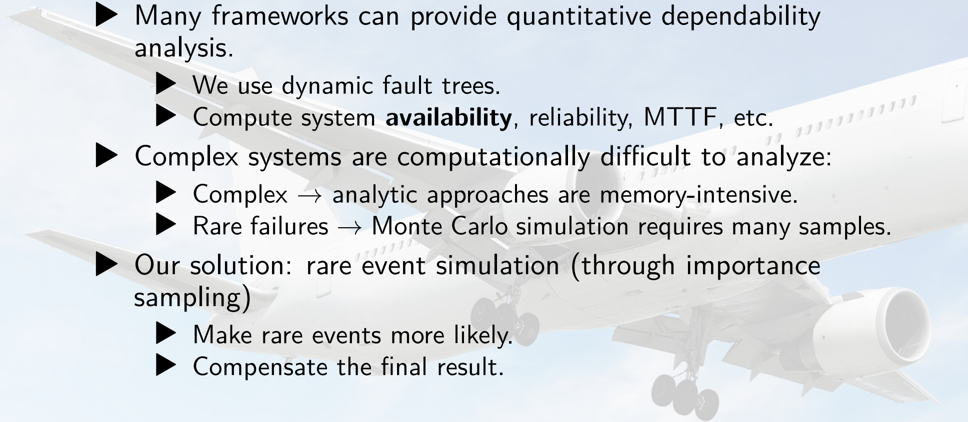

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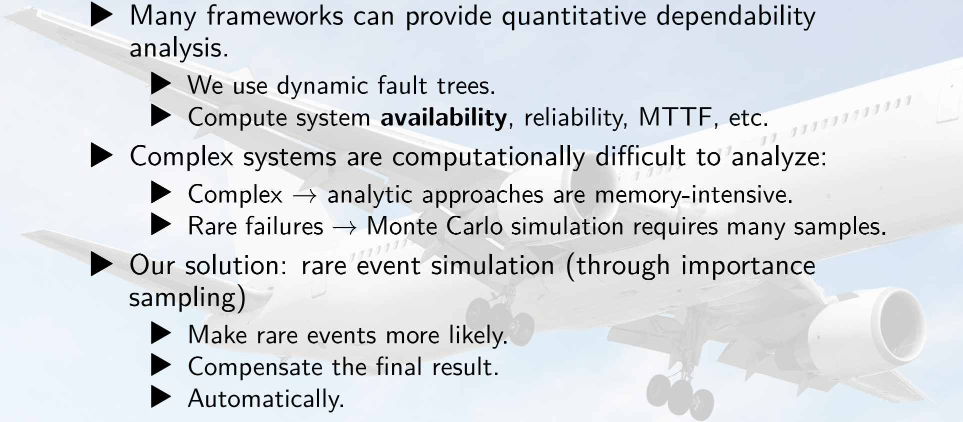

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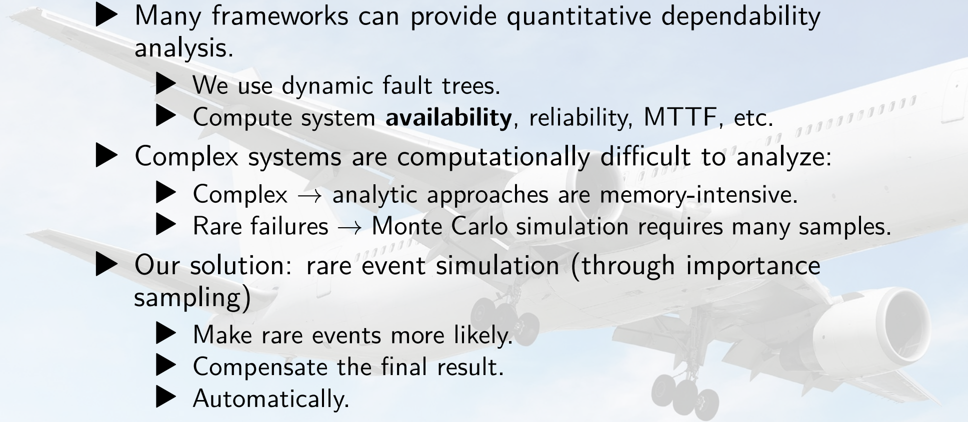

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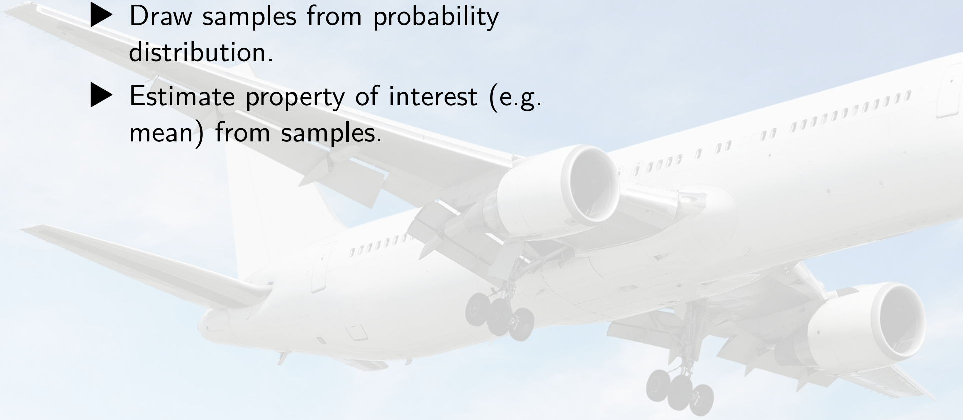
- **Rare event simulation + dynamic fault trees → Faster/more accurate fault tree simulation.**

Monte carlo simulation

- Draw samples from probability distribution.
- Estimate property of interest (e.g. mean) from samples.



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- Drawback: For improbable events, many samples are needed.

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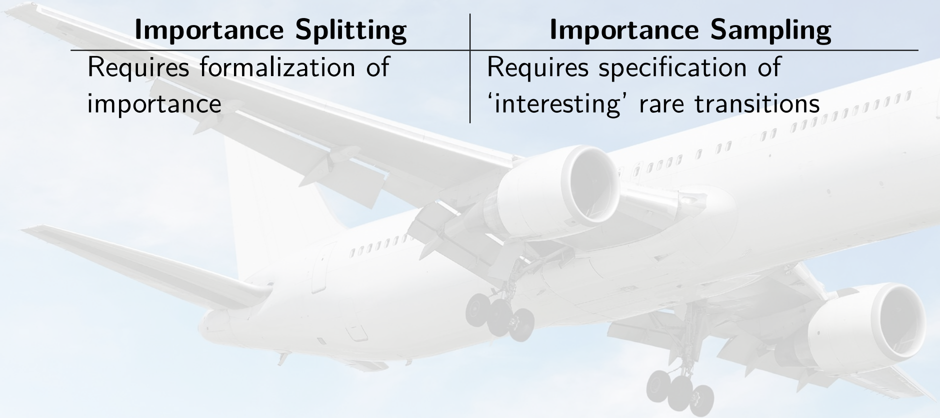
- Yields more accurate results and/or needs fewer samples.

Comparison of RES techniques



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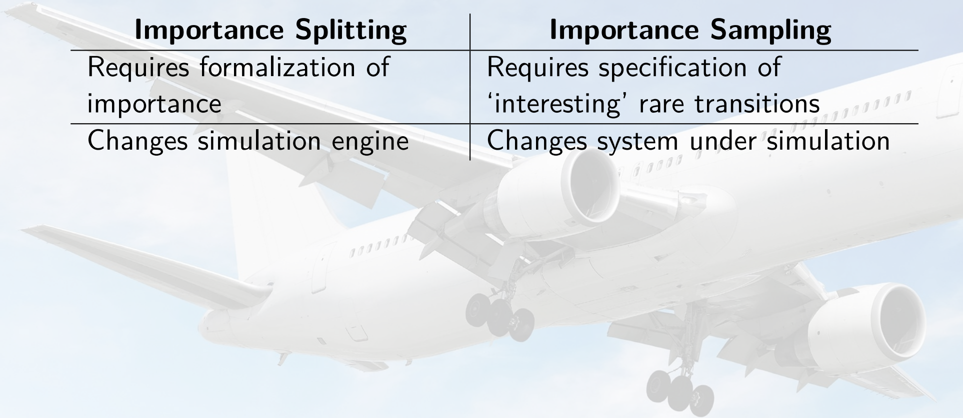

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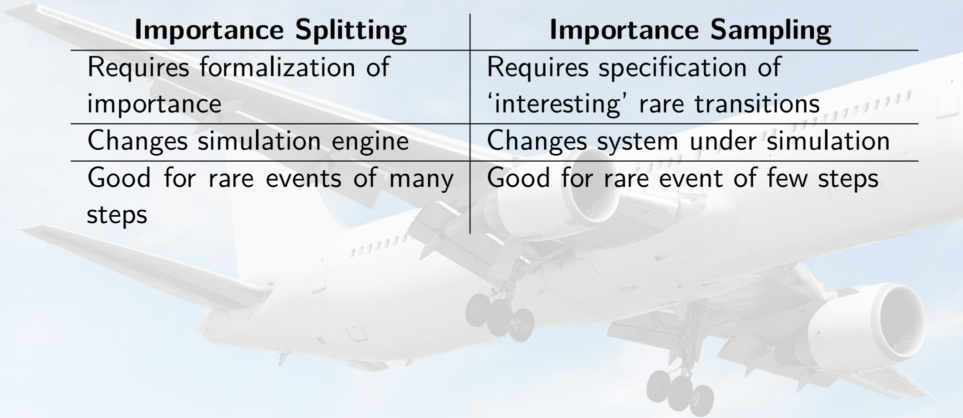

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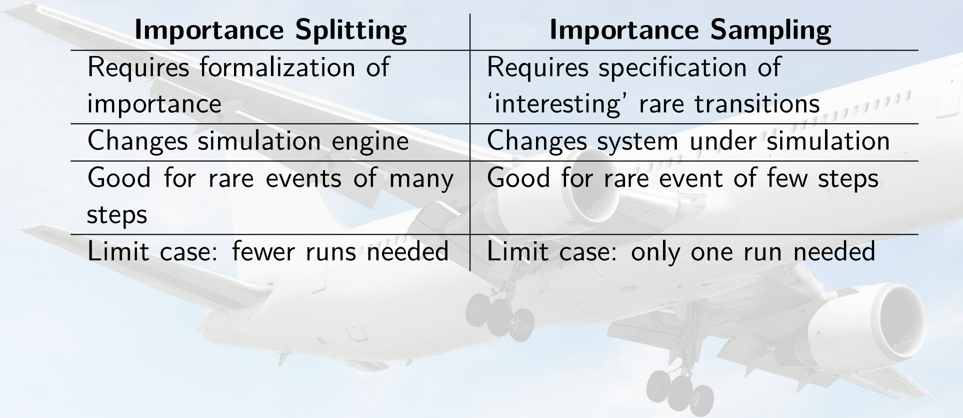

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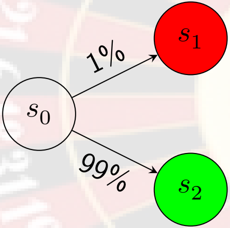
We use importance sampling as our system reaches the rare event after only a few, low-probability transitions. Such models provide few points to split the samples.

Importance sampling of Markov chains: an example



- Probability of red state?

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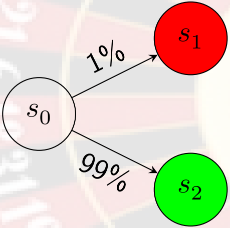

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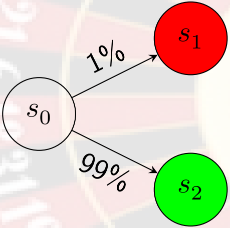

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- Estimate 2 (100 runs): 1%

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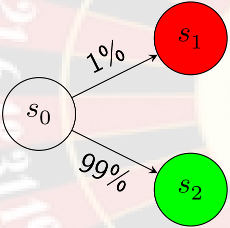

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- Estimate 3 (100 runs): 1%
- Estimate 4 (100 runs): 1%
- Estimate 5 (100 runs): 0%
- Estimate 6 (100 runs): 0%
- Estimate 7 (100 runs): 2%

Importance sampling of Markov chains: an example

Make bad state 10 times more likely:

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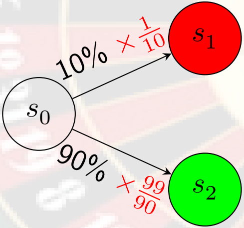

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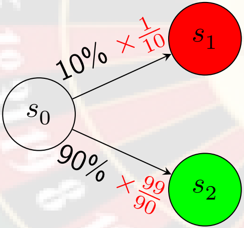

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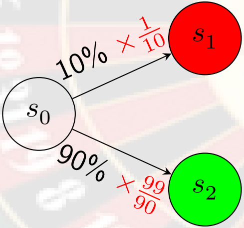

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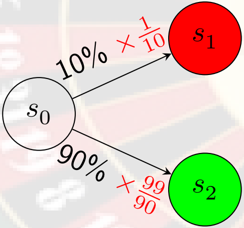

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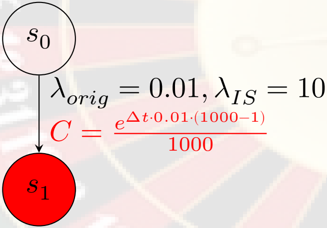
- Estimate 5 (100 runs): $13 \cdot \frac{1}{10} = 1.3\%$
- Estimate 6 (100 runs): $8 \cdot \frac{1}{10} = 0.8\%$
- Estimate 7 (100 runs): $14 \cdot \frac{1}{10} = 1.4\%$

Importance sampling of CTMCs: overcompensating

Increasing the rare event too much:

- Probability of reaching red state within 1 time unit?



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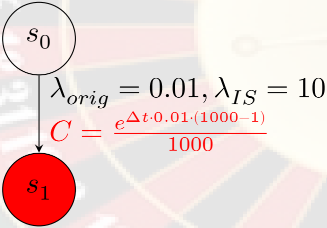

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$C = \frac{e^{\Delta t \cdot 0.01 \cdot (1000-1)}}{1000}$

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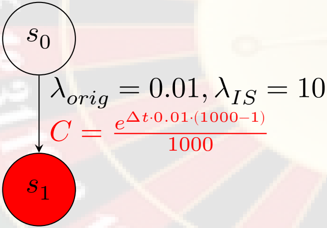

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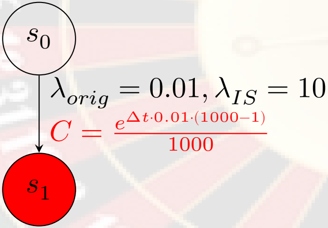

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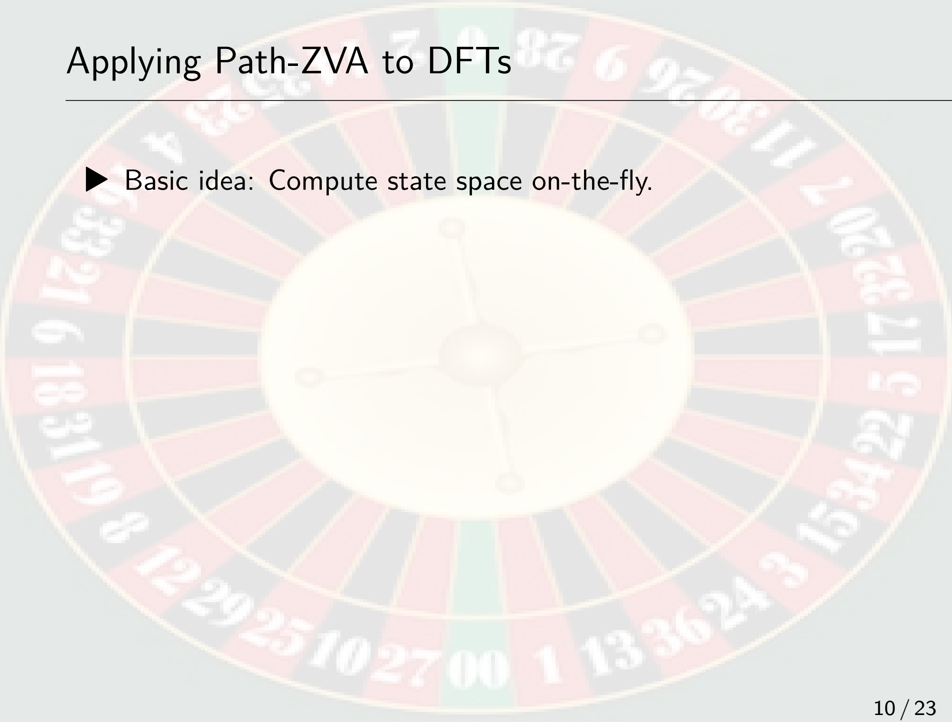
- Transition rates parameterized as $r \cdot \epsilon^n$ with $0 < \epsilon << 1$ to indicate ‘rareness’.

Applying Path-ZVA to DFTs



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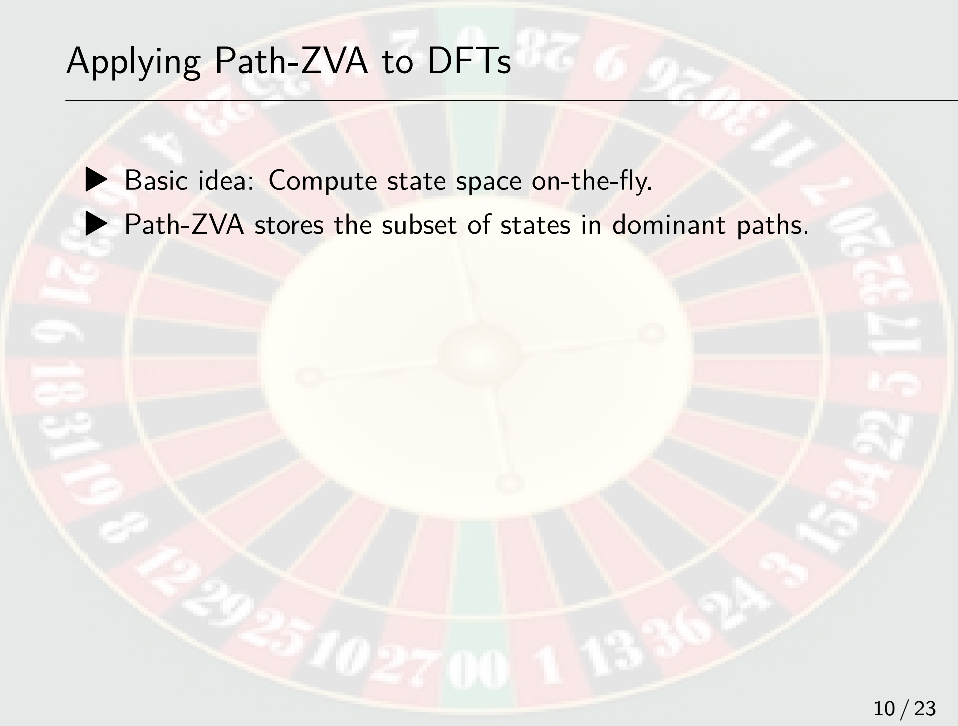

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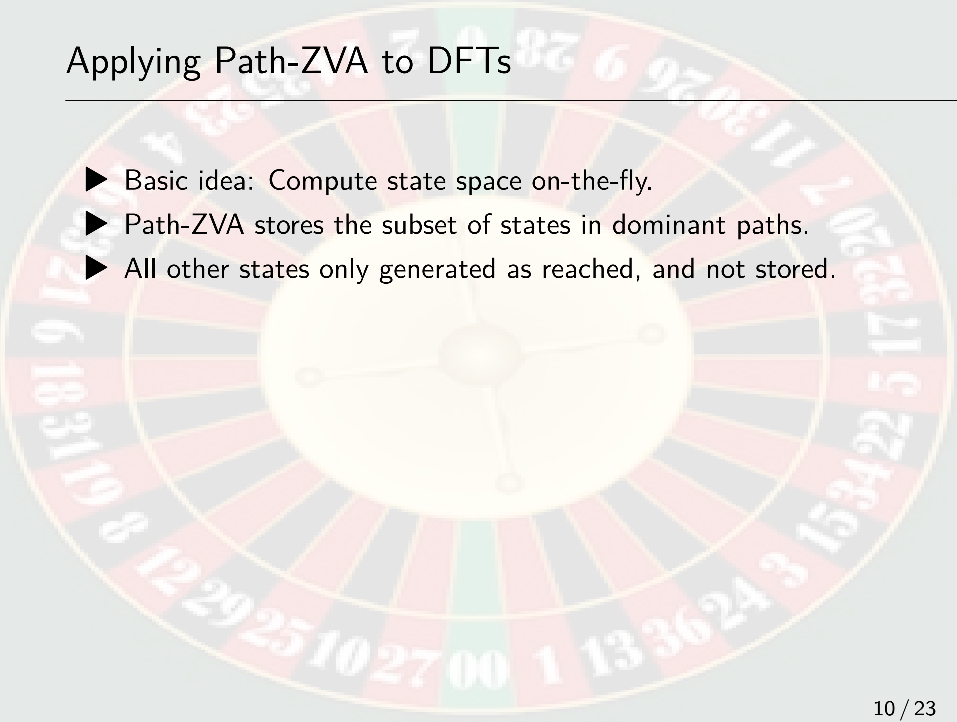

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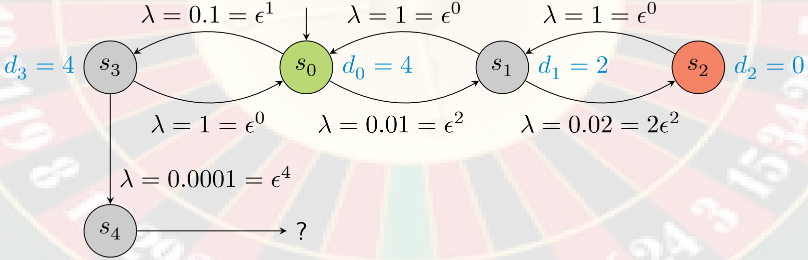
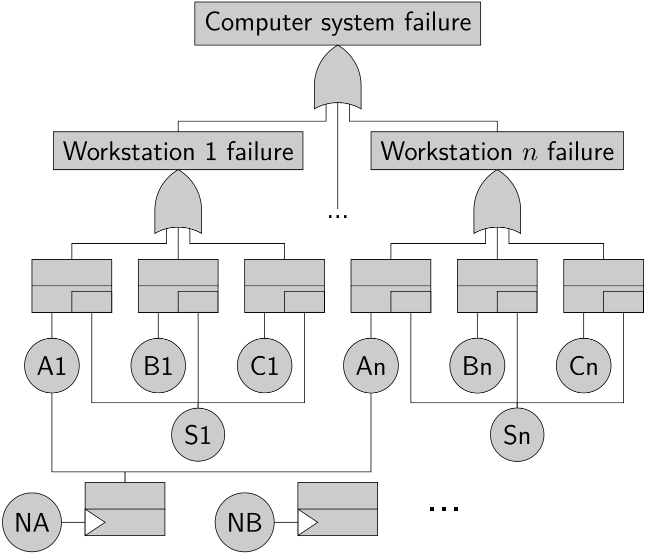


DFT example


Computer system failure
Workstation 1 failure
Workstation n failure
...
A1
B1
C1
An
Bn
Cn
S1
Sn
NA
NB
...

From I/O-IMCs to Markov Chains

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- DFTCalc produces I/O-IMCs.

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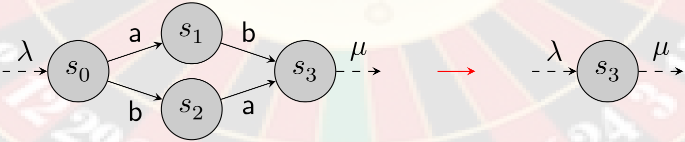
- How to resolve nondeterminism?
- Generally, the nondeterminism is spurious:
  - Regardless of choices, you end up in the same Markovian state.
- We verify that it is spurious, then collapse the interactive transitions before the next state:

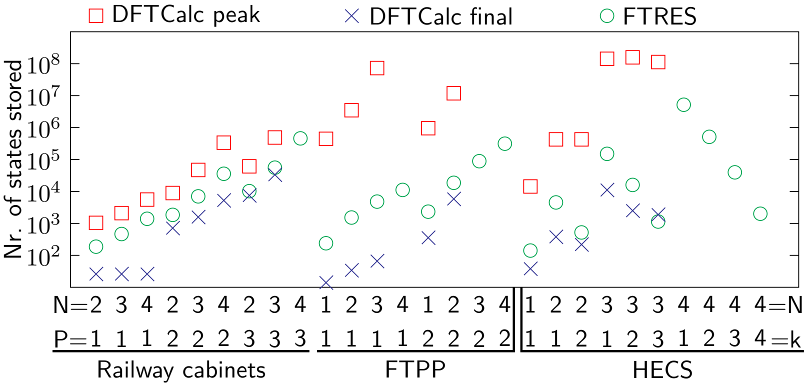


Results: Unavailability

Exact result for DFTCalc, 95% confidence for others:

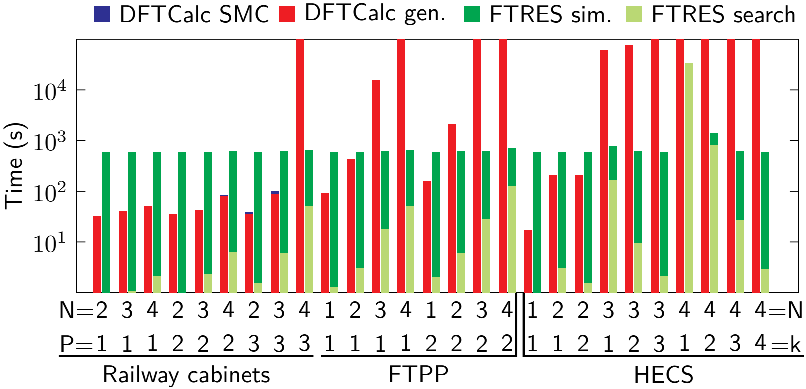
			Unavailability		
	N	**P**	**DFTCalc**	**FTRES**	**MC**
FTPP	1	1	$2.18303 \cdot 10^{-10}$	$[2.182; 2.184] \cdot 10^{-10}$	—
	4	1	—	$[2.226; 2.232] \cdot 10^{-10}$	—
	1	2	$1.76174 \cdot 10^{-20}$	$[1.761; 1.762] \cdot 10^{-20}$	—
	4	2	—	$[1.760; 1.763] \cdot 10^{-20}$	—
	N	**k**	**DFTCalc**	**FTRES**	**MC**
HECS	1	1	$4.12485 \cdot 10^{-5}$	$[4.124; 4.126] \cdot 10^{-5}$	$[4.079; 4.156] \cdot 10^{-5}$
	2	1	$3.02469 \cdot 10^{-9}$	$[3.022; 3.026] \cdot 10^{-9}$	$[0; 9.040] \cdot 10^{-9}$
	2	2	$8.24940 \cdot 10^{-5}$	$[8.247; 8.251] \cdot 10^{-5}$	$[8.218; 8.338] \cdot 10^{-5}$
	4	1	—	$[3.902; 4.364] \cdot 10^{-17}$	—
	4	2	—	$[1.239; 1.252] \cdot 10^{-12}$	—
	4	3	—	$[1.813; 1.818] \cdot 10^{-8}$	$[0; 8.352] \cdot 10^{-9}$
	4	4	—	$[1.648; 1.651] \cdot 10^{-4}$	$[1.621; 1.657] \cdot 10^{-4}$

Overall results: State space



- FTRES always below DFTCalc maximal state space size.
- FTRES computes results where DFTCalc does not.

Overall results: Speed



- FTRes and MC spend a constant 10 mins. simulating.
- Simulation time mostly dominates state-space exploration.
- Several DFTCalc experiments ran longer than 24 hours.

WIP: Extending to time-bounded reachability

- Standard Path-ZVA leaves total exit rates unchanged.

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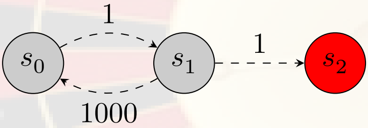
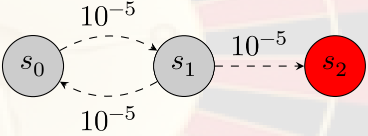


Rarity due to
relative probability



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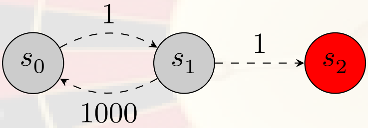

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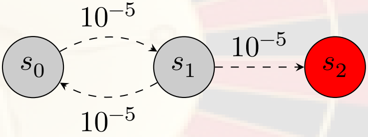

Rarity due to
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- Path-ZVA helps the first case, not the second.

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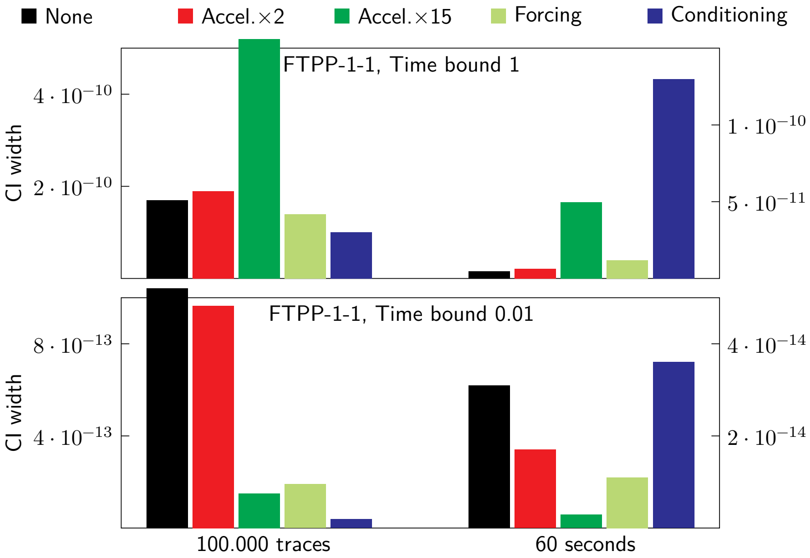
- *Forcing*: Sample transition times conditional on time bound.
    - Pretty good performance in general.
    - Guaranteed better than no forcing (sort of).
    - Moderate loss of performance.

Preliminary results on time-bounded reachability



Preliminary results on time-bounded reachability

Results for FTPP-1-1, other models behave similarly:

Time bound	Forcing /conditioning	CI width 100.000 sims	Simulation time	CI width 60s sim.
1	None	$1.7 \cdot 10^{-10}$	0.16	$4.8 \cdot 10^{-12}$
1	Accel. × 2	$1.9 \cdot 10^{-10}$	0.15	$6.4 \cdot 10^{-12}$
1	Accel. × 15	$1.5 \cdot 10^{-9}$	0.15	$5.0 \cdot 10^{-11}$
1	Forcing	$1.4 \cdot 10^{-10}$	0.60	$1.2 \cdot 10^{-11}$
1	Conditioning	$1.0 \cdot 10^{-10}$	70	$1.3 \cdot 10^{-10}$
0.01	None	$2.1 \cdot 10^{-12}$	0.05	$3.1 \cdot 10^{-14}$
0.01	Accel. × 2	$9.8 \cdot 10^{-13}$	0.08	$1.7 \cdot 10^{-14}$
0.01	Accel. × 15	$1.5 \cdot 10^{-13}$	0.07	$2.9 \cdot 10^{-15}$
0.01	Forcing	$1.9 \cdot 10^{-13}$	0.36	$1.1 \cdot 10^{-14}$
0.01	Conditioning	$3.9 \cdot 10^{-14}$	53	$3.6 \cdot 10^{-14}$

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- Fixed acceleration good but fiddly.
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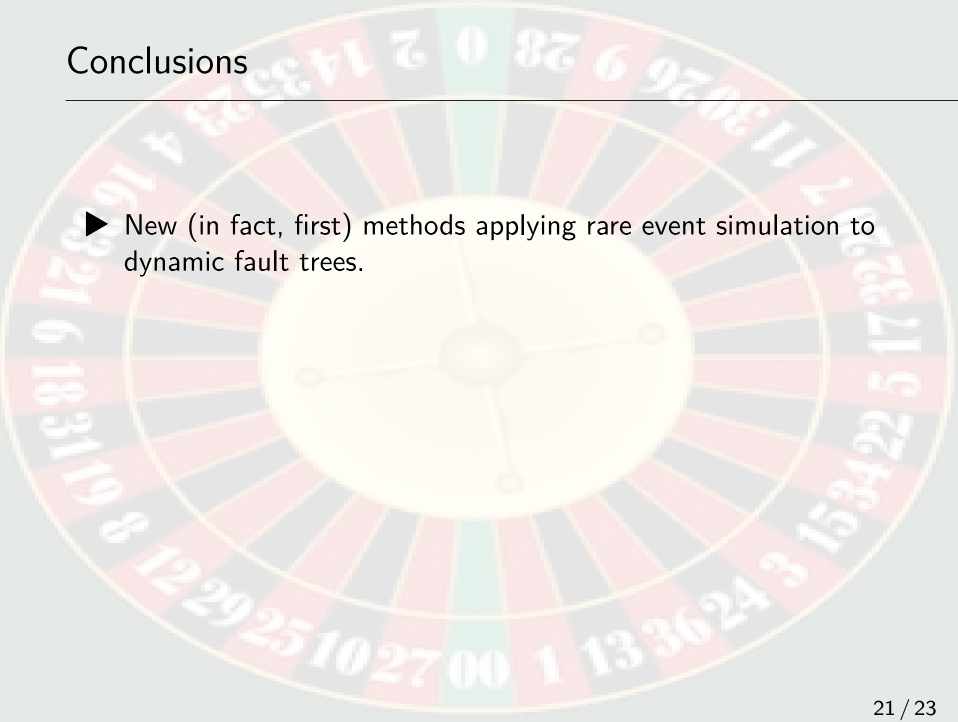
  - Bad choices can result in biased-looking estimators.
- Forcing generally works well.
  - Mostly independent of model rates and property.

Conclusions



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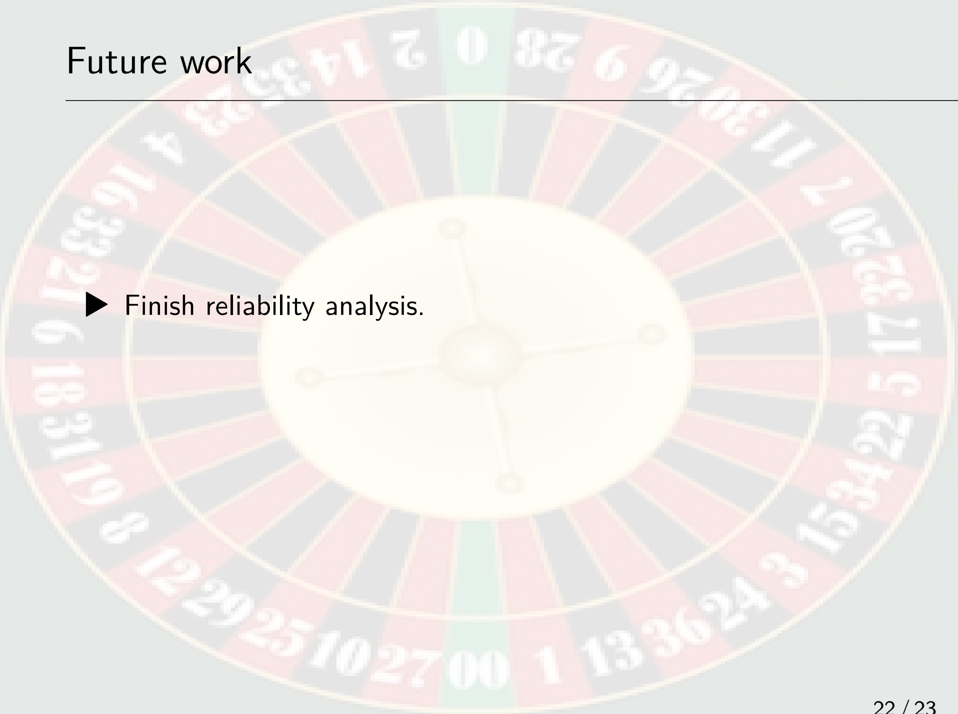
- Analysis of availability and reliability, with reliability still being improved.

Future work



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- Limited nondeterminism to cover full set of DFTs

Thank you for your attention.

Questions?