The state of the art in fault tree modeling and analysis

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November 11, 2014

Outline

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Introduction to fault trees

- Developed in 1961 by Nuclear Regulatory Agency
- Question: How reliable is your system?
- Now used by:

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- Question: How reliable is your system?
- Now used by:

Why fault trees?

- Some things really should not fail
- Risk assessment is sometimes mandatory
    - Probability of catastrophic failures?
    - Biggest risk factors?
    - Possible mitigations?

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- Some things really should not fail

  Reliability: Probability of failing within given time

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  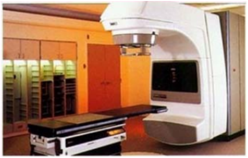

Why fault trees?

- Some thing should not fail for long

  Availability Proportion of time in functioning state

amazon.com

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What do we want to know?

Qualitative:

- Insight into biggest risks
- Relatively fast to perform
- Easy to understand
- Limited information

Quantitative:

- Quantify total risk
- Quantify effect of mitigation
- Time consuming
- Hard to estimate numbers

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- **Cut sets**: Sets of components causing failure
  Example: Airplane fails when both engines fail

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Qualitative:

- **Cut sets**: Sets of components causing failure
  Example: Airplane fails when both engines fail
- **Common cause failures**: Multiple failures with one cause
  Example: Redundant computers running same program

What to we want to know?

Quantitative:

- **Reliability** $\equiv$ Probability of failure within time t
  Example: Probability of containment failure within 25 year nuclear plant lifetime

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  Example: Amazon EC2 cloud offers SLA of 99.95% uptime

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  Example: Probability of containment failure within 25 year nuclear plant lifetime
- **Availability** $\equiv$ Proportion of time (in $[0, \infty\rangle$ or $[0, t]$) spent not failed
  Example: Amazon EC2 cloud offers SLA of 99.95% uptime
- **MTBF** $\equiv$ Expected time between two successive failures (in finite or infinite horizon)
  Example: How frequently will my car break down?

What to we want to know?

Quantitative:

MTTF $\equiv$ Expected time between system becoming functioning and failing
Example: How long will my car run after a service?

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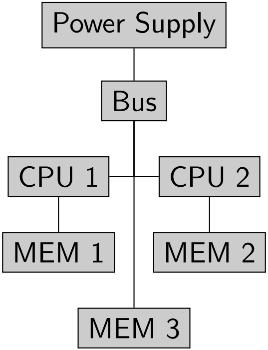
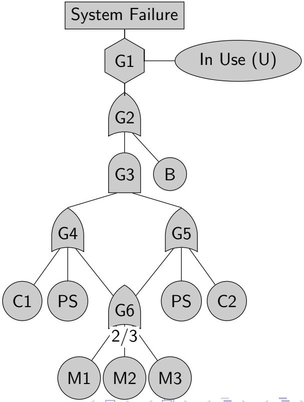
MTTFF $\equiv$ Expected time before first failure
Example: How long will my new car without failing?

ENF $\equiv$ Expected number of failures
Example: How many switches will fail in the country per year?

Fault tree example





- Redundant CPUs
- 1 shared spare memory unit

Fault tree summary

- Graphical model
- Widely used in industry
- Used for reliability, availability and safety analysis

Fault tree elements

- Basic events (leaves)
- Intermediate Events (gates)
- Top (Level) Event (gate)
- DAG, but often shows as tree with duplicated events

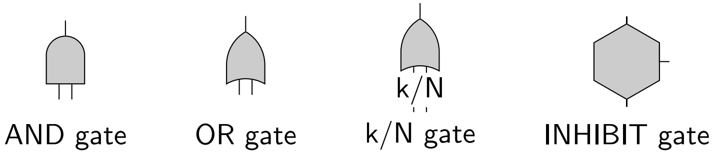
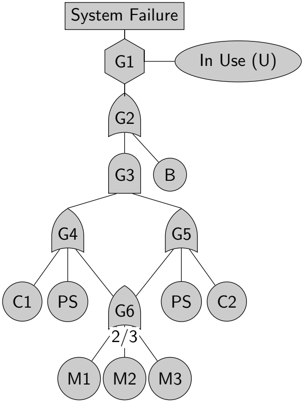


Figure: Images of the gates types in a static fault tree

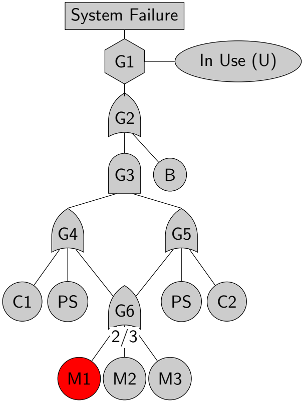
Example of fault tree failure propagation

- No failures



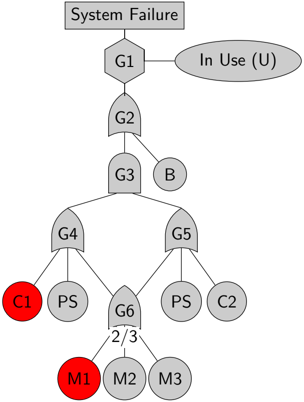
Example of fault tree failure propagation

- Failure of M1



Example of fault tree failure propagation

- Failure of M1
- Failure of C1



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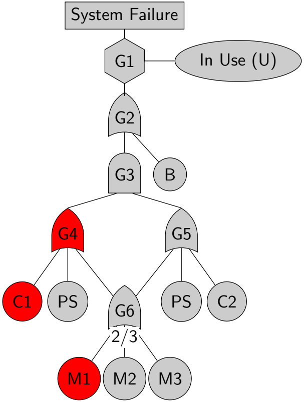
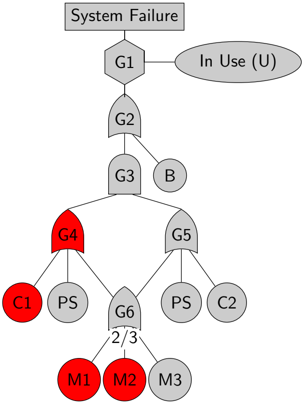


Example of fault tree failure propagation

- Failure of M1
- Failure of C1
- Failure of M2



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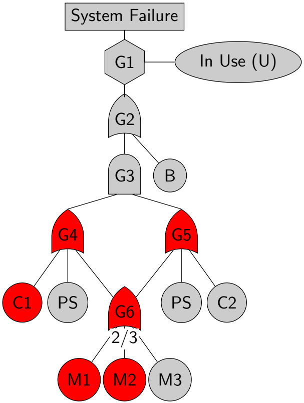

Example of fault tree failure propagation

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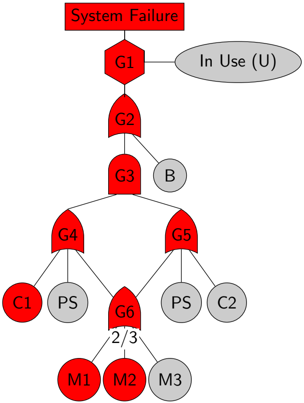

Outline

Measures of interest

Qualitative:

- Cut sets
- Path sets
- Common Cause Failures

Quantitative:

- Reliability
- Availability
- MTBF/MTTF/MTTFF
- Expected number of failures
- importance values

Qual. analysis: Cut sets

- Set of components causing failure
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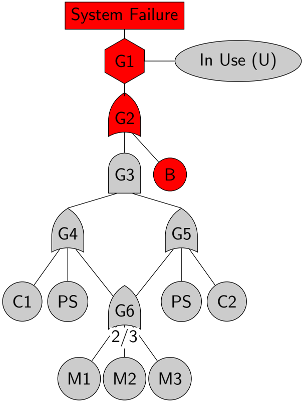

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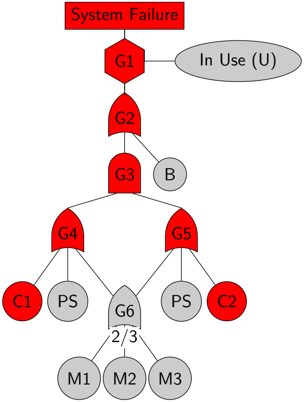
- Set of components causing failure
- Usually minimal cut sets
- Small cut sets like candidates for system improvement
- Examples: {U,B}, {U,C1,C2}, . . .



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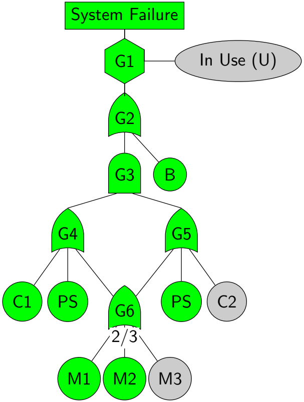
- Example: {B,PS,C1,M1,M2}



Outline

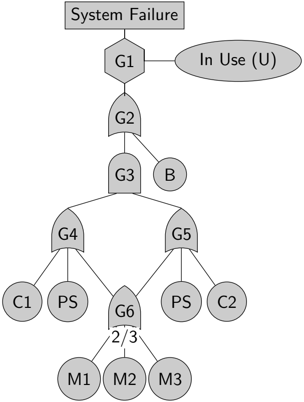
- Cut sets: Boolean manipulation
- Cut sets: Binary Decision Diagrams
- Common cause failures

Cut set analysis: Boolean manipulation

- Use boolean algebra to construct DNF
- Bottom-up: Start with leaves
- Top-down: Start with root

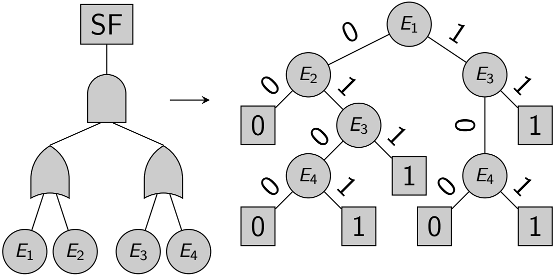
Cut set analysis: Boolean manipulation

- Use boolean algebra to construct DNF
- Bottom-up: Start with leaves
- Top-down: Start with root
- Example (top-down):
- $G1 = U \wedge G2$ and $G2 = B \vee G3$
- $G1 = U \wedge (B \vee G3)$
- $G1 = (U \wedge B) \vee (U \wedge G3)$



Cut set analysis: Binary Decision Diagrams

- DAG representing boolean function
- Leaves are 0 or 1
- All paths from the root have the same variable ordering



Common cause failures

- Simultaneous failures of multiple components
- Examples: fire, earthquake, wear of identical components
- Cannot be derived from FT structure
- Expert insight to determine CCF within cut sets

Outline

- Fault tree types
- Bottom-up method
- Rare-event approximation
- Bayesian networks
- Monte Carlo Simulation

Fault tree types

Time:

- Discrete-time (one-shot)
- Continuous-time without repairs
- Continuous-time with independent repairs

Failure distributions:

- Single probability (discrete-time only)
- Exponential distribution
- Arbitrary distribution

Quant. analysis: Bottom-up method

- When no events are shared:
- $\mathbb{P}[X_{AND}(X_1, X_2, \cdots, X_n) = 1]$
- $= \mathbb{P}[X_1 = 1 \land X_2 = 1 \land \cdots \land X_n = 1]$
- $= \mathbb{P}[X_1 = 1]\mathbb{P}[X_2 = 1] \cdots \mathbb{P}[X_n = 1]$
- Likewise for other gates
- Same for availability

Quant. analysis: Rare event approximation

- Assuming failures are infrequent (e.g. 10^{-9})
  - Approximate using $\mathbb{P}(A \vee B) \approx \mathbb{P}(A) + \mathbb{P}(B)$
  - Sum unavailabilities or unreliabilities of cut sets
- Can be made exact using inclusion-exclusion principle:
  - $\mathbb{P}(A \vee B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \wedge B)$

Quant. analysis: Bayesian Networks

- General technique used in many probabilistic analyses
- Express fault tree in conditional probabilities

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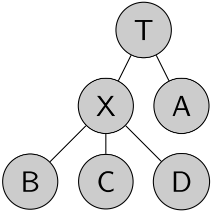
- General technique used in many probabilistic analyses
- Express fault tree in conditional probabilities
- Example (A or (B and C and D)):



$$\mathbb{P}(T = 1 | A = 1 \lor X = 1) = 1$$
$$\mathbb{P}(A = 1) = 0.1$$
$$\mathbb{P}(X = 1 | B = C = D = 1) = 1$$
$$\mathbb{P}(B = 1) = 0.3$$
$$\mathbb{P}(C = 1) = 0.4$$
$$\mathbb{P}(D = 1) = 0.1$$

Quant. analysis: Bayesian Networks

Advantages:

- Inference using existing tools
- Allows diagnosis
- FT structure persists into model
- Easy to extend with e.g. probabilistic gates

Disadvantages:

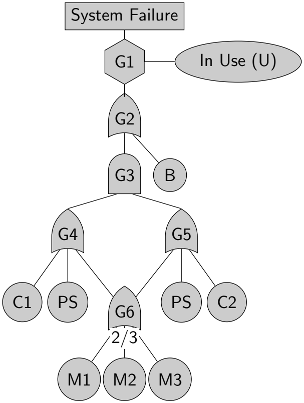
- Conditional probability table exponentially large in nr. of inputs

Quant. analysis: Monte Carlo simulation

- Simulation used in many applications
- Sample failures or failure times, and repair times if needed
- Propagate failures through the tree at every failure or repair
- Track measure of interest through repeated simulations

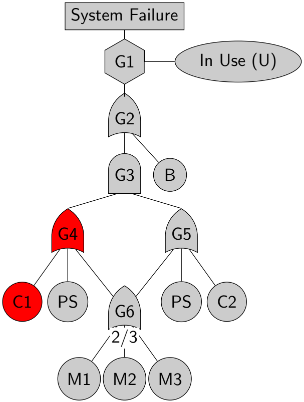
Monte Carlo Simulation example

- All BEs have failure probability 0.2
- Runs: 0
- Failures: 0
- Estimated reliability:



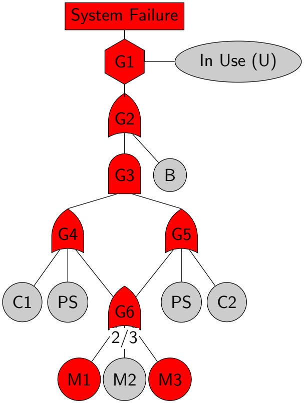
Monte Carlo Simulation example

- All BEs have failure probability 0.2
- Runs: 1
- Failures: 0
- Estimated eliability: 1



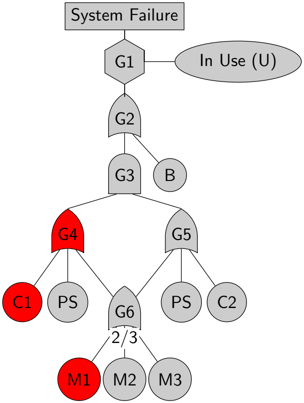
Monte Carlo Simulation example

- All BEs have failure probability 0.2
- Runs: 2
- Failures: 1
- Estimated eliability: 0.5



Monte Carlo Simulation example

- All BEs have failure probability 0.2
- Runs: 3
- Failures: 1
- Estimated eliability: 0.666



Summary

Quantitative analysis techniques:

- Bottom-up method
- Rare-event approximation
- Bayesian networks
- Monte Carlo Simulation

Other techniques:

- Algebraic analysis
- Algebraic approximation

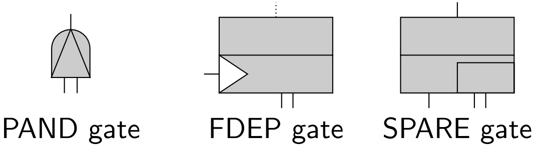
Outline

Shortcomings of fault trees

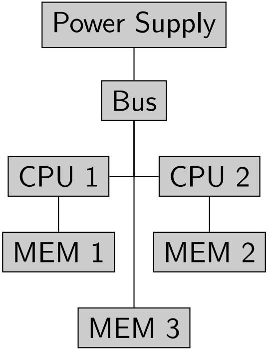
- No information about failure sequences
- Poor modeling of shared spare components
- Dependencies cause large trees
- One solution: Dynamic fault trees (DFTs)

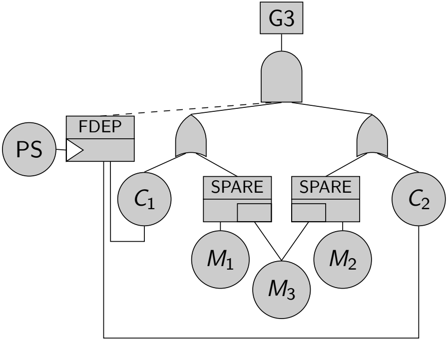
Dynamic fault trees

Three new gates:



DFT Example


Power Supply
Bus
CPU 1
CPU 2
MEM 1
MEM 2
MEM 3


G3
FDEP
PS
C_1
SPARE
SPARE
C_2
M_1
M_2
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Outline

- Cut sets
- Cut sequences

Qualitative analysis

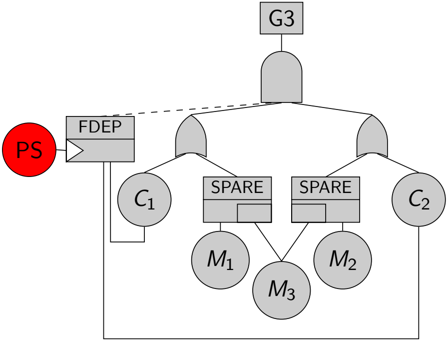
Cut sets for DFTs:

- Failures of cut sets CAN cause system failures, depending on ordering
  - Due to shared spares, failure not always caused by cut sets
- Convert DFT into SFT, by replacing:
  - PAND $\to$ AND
  - SPARE $\to$ AND
  - FDEP $\to$ OR

DFT cut sets Example

Example cut sets:

- {PS}



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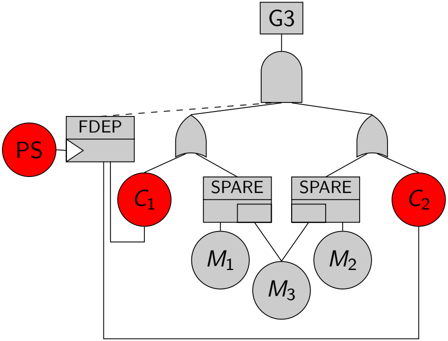

DFT cut sets Example

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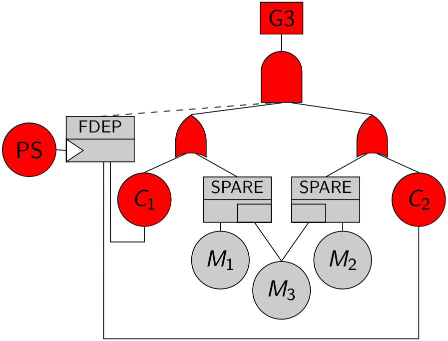
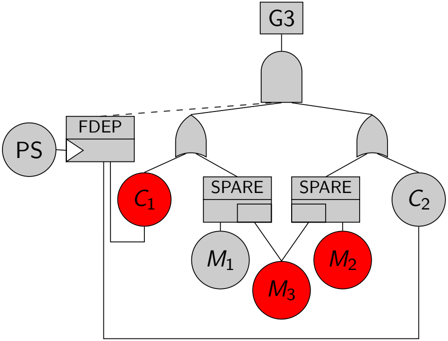


DFT cut sets Example

Example cut sets:

- {PS}
- {C1,M1,M2}



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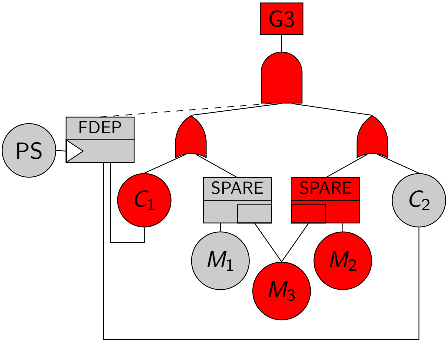
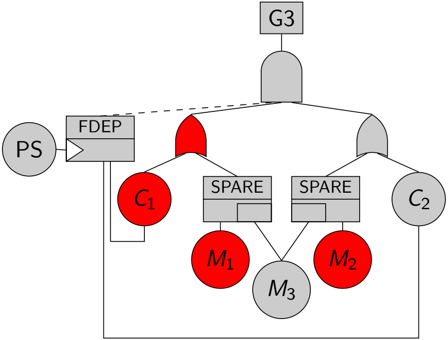


DFT cut sets Example

Example cut sets:

- {PS}
- {C1,M2,M3}
- NOT {C1,M1,M2}



Qualitative analysis

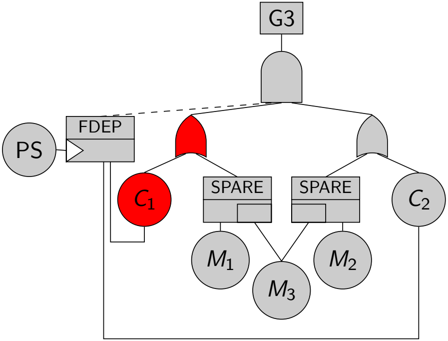
Cut sequences:

- Like cut sets, but include sequence information
- Failure of a cut sequence always causes system failure
- Any system failure is caused by a cut set failure

DFT cut sequences Example

Example cut sequence:

- $\langle C1, M1, M2 \rangle$



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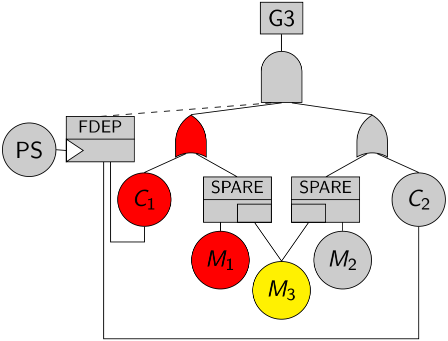

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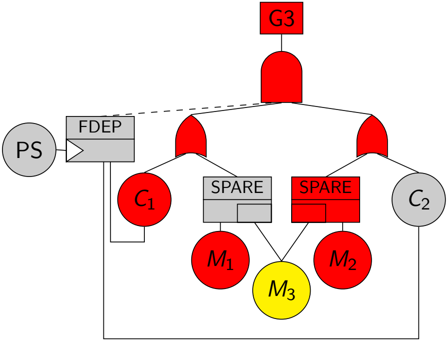
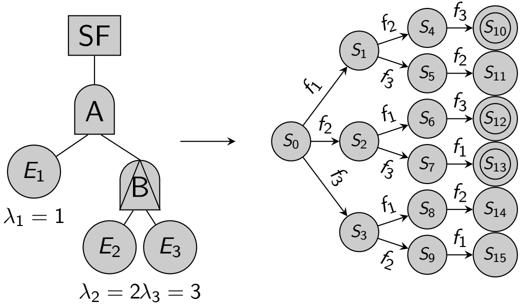


Outline

- Markov analysis
- I/O IMC

Quant. analysis: Markov chain

Analysis by markov chain:



Quant. analysis: Markov chain

Advantages:

- Exact semantics
- No nondeterminacy
- Reuse of existing modelcheckers (PRISM, etc.)

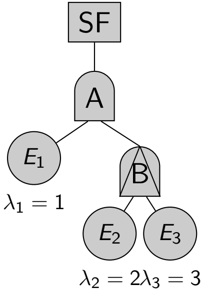
Disadvantages:

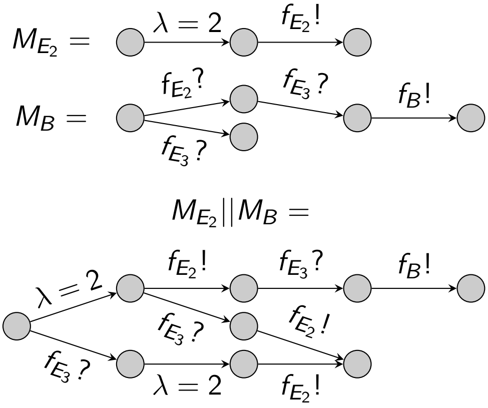
- Semantics are ca. 20 pages long
- Combinatorial explosion

Quant. analysis: Compositional Markov Analysis

- *Input/Output Interactive Markov Chains* exist of gates and basic events
- Input/Output signals allow parallel composition
- Models of FT elements are composed into one large model

Quant. analysis: I/O IMC example





Quant. analysis: Compositional Markov Analysis

Advantages:

- Semantics easier to understand
- Intermediate minimization reduces state-space explosion
- Easy to add new gates or events
- Can model nondeterminacy

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Note: This is the approach used in DFTCalc and the ArRangeer project

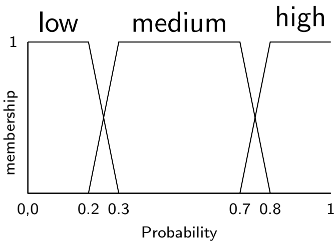
Other quantitative analysis methods

- Petri Nets
- Dynamic Bayesian Networks
- Modularization of static and dynamic subtrees
- Monte Carlo Simulation

Outline

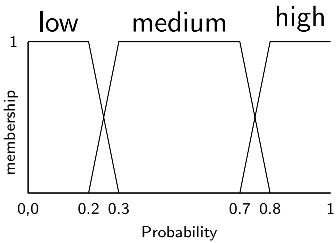
Fuzzy numbers

- Uncertainty and variation in BE probabilities
- Expert judgement not exact
- Possible solution: BE probabilities in fuzzy sets
- Several frameworks for computations on fuzzy numbers
- Can compute same measures as for non-fuzzy FTs.



Fuzzy computations

- Combine fuzzy sets using mathematical operations
- Problem: probability distribution unknown
- Various assumptions exist, partly for computational efficiency
- Example: medium + medium = (not low, maybe medium, likely high)



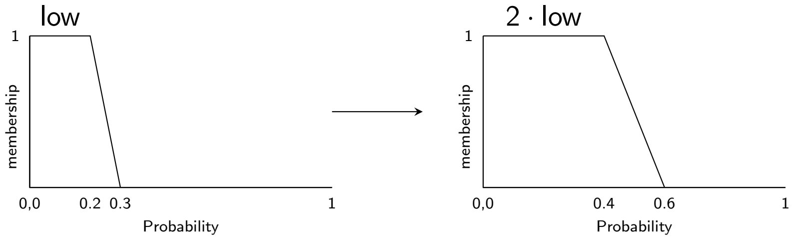
Fuzzy computations

Extension principle:

- Commonly used method
- Given:
  - $f : X \mapsto Y$ for *crisp* (i.e. non-fuzzy) sets
  - A fuzzy set A with membership function $\mu_A : X \mapsto [0, 1]$
- Define $B = f(A)$:
  - $\mu_B(y) = \max_{x=f^{-1}(y)} \mu_A(x)$

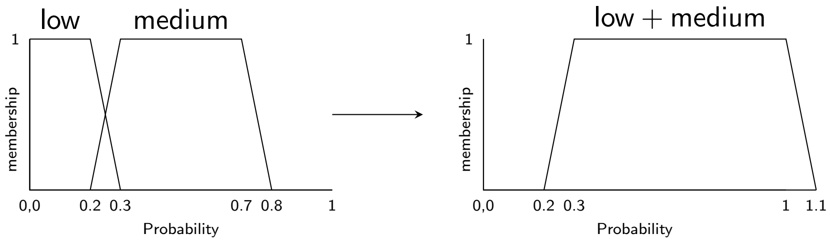
Fuzzy computations example

- Consider fuzzy numbers (fuzzy subsets of $\mathbb{R}$)
- Let $f(x) = 2x$, $A = \text{low}$, $B = f(A)$
- Then $\mu_B(0.4) = 1$ since $\mu_A(0.2) = 1$ and $2 \cdot 0.2 = 0.4$



Fuzzy addition

- $\mu_{A+B}(z) = \max_{z=x+y}(\min\{\mu_A(x), \mu_B(y)\})$
- Example: $\mu_{\text{low+medium}}(0.9) = 1$



Fuzzy arithmetic

- Problem: Fuzzy arithmetic does not return original values
- Various methods to may fuzzy sets back onto descriptors
- In practice: expert judgement

Other uncertain FTs

- ‘Intuitionistic fuzzy set theory’: Membership function uncertain
- Probability distribution for BE failure rates
- Multi-state BE with uncertain states
- Normal distribution approximation

Outline

FTs with dependent events

- Normal FTs assume independent BEs
- Not always realistic ('valve stuck open' and 'valve stuck closed' are not independent)
- Component failures and degradation may propagate

Extended fault trees

- Components have multiple states (between ‘failed’ and ‘perfectly working’)
- Component state can affect other component failure rates
- Gates allow for different combinations of states
- Textual DSL needed for specification
- Only quantitative continuous-time analysis defined

Boolean Driven Markov Processes

- BEs and gates represented as multiple Markov Processes (MPs)
- States in the MPs can trigger other elements to switch MPs
- Applications: Changing failure rates, multistate components, new gates
- Disadvantage: Harder to quickly oversee
- Only quantitative continuous-time analysis defined

Other dependent event extensions

- Multiple FTs for different failure modes
- Specifying mutually exclusive events
- Replace BEs by Petri nets

Outline

Repairable fault trees

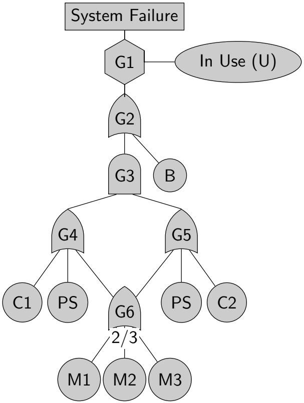
- Simple repair model: Simultaneous independent repairs
- Problem: Limited resources for repairs in real life
- Problem: Hidden failures
- Solution method: Repairable Fault Trees

Repairable fault trees

- Add *Repair Boxes* to tree (now becomes cyclic)
- Repair box has one input: repair starts when input fails
- Repair box specifies multiple components to repair
- Repair policy determines how repairs procees (simultaneous, sequential, combination, etc.)
- Qualitative analysis (cut sets) possible but less useful
- Several quantitative analysis techniques defined

Example RFT

- Start repair when G4 fails with priorities:
  1. PS
  2. M1, M2, M3
  3. C1
- Similar for G5
- Repair B when G2 fails



Outline

Fault trees with temporal properties

- Static FTs do not consider timing information
- DFTs are one approach to include them, others exist

FTs with temporal gates

Add new gate types:

- AND-THEN gate: Requires one event ‘immediately after’ another
  - Formal description with informal predicate
  - Only qualitative analysis defined (extended cut sequence)

FTs with temporal gates

Add new gate types:

- AND-THEN gate: Requires one event 'immediately after' another
  - Formal description with informal predicate
  - Only qualitative analysis defined (extended cut sequence)
- POR: fail when first input fails before others
- SAND: fail on simultaneous failure of all inputs
  - PAND + POR + SAND strictly more expressive than AND-THEN gate
  - Only qualitative analysis defined
  - Quantitative analysis seems easy to add

FTs with temporal logic

Several approaches add temporal logics to FTs:

- Cause-consequence gates
  - Allows indeterminate delays
  - Qualitative analysis for failure-preventing cut sets
  - No other analysis possible
- Duration calculus
  - Calculus allows reasoning about delays
  - Not proven decidable
  - No automated analysis available
- Propositional Linear Temporal Logic
  - Adds single-input gates like PREV and SOMETIME-PAST
  - Qualitative analysis defined
  - Quantitative analysis probably also possible

Outline

State-Event Fault Trees

- Practical system failures are sometimes state-dependent
- Especially true of computer software
- SEFT combine state machines with FT gates
- State transitions cause events
- Events and states are combined in gates
- Events can cause state transitions
- Later additions include delays, probabilistic gates
- Quantitative analysis by Petri Nets

Outline

Conclusion